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О МОДЕЛИРОВАНИИ ПРОИЗВОДСТВЕННОГО ПРОЦЕССА С ИСПОЛЬЗОВАНИЕМ ПРЕДМЕТНО-ТЕХНОЛОГИЧЕСКОГО И ПОТОКОВОГО УРОВНЕЙ ОПИСАНИЯ

С использованием статистического подхода, который широко распространен в естественных науках, представлена модель производственного процесса производственно-технической системы с поточным способом организации производства. Состояние макропараметров производственного процесса производственно-технической системы задается множеством состояний предметов труда. Состояние отдельного предмета труда задано точкой в фазовом технологическом пространстве. Введена функция распределения предметов труда по состоянию и записано кинетическое уравнение для функции распределения предметов труда по состояниям технологической обработки. Записана замкнутая система динамических уравнений (уравнений баланса), описывающая поведение во времени значений первых моментов функции распределения, которые характеризуют плотность распределения межоперационных заделов и темп обработки предметов труда по технологическим операциям.

Ключевые слова: производственная линия; предмет труда; поточная линия; параметры состояния поточной линии; технологическая позиция; переходной период; системы управления производством; кинетическое уравнение; массовое производство; незавершенное производство; PDE-модель производства.

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ABOUT SIMULATION OF PRODUCTION WITH THE USE OF SUBJECT-TECHNOLOGICAL AND FLOW LEVELS OF DESCRIPTION

Using the statistical approach, which is widespread in the natural sciences, a model of the production process of the technological system with a streamlined method of organizing production is presented. The state of macroparametres of the production process of the technological system is determined by the set of states of subjects of labor. The state of a particular subject of labor is given by a point in the phase of technological space. The function of distribution of subjects of labor by state is introduced, and the kinetic equation for the distribution function of subjects of labor according to the conditions of technological processing is written.

A closed system of dynamic equations (balance equations), that describes the behavior in time of the meanings of the first moments of the distribution function, which characterize the distribution density of interoperational stocks and the rate of processing of subjects of labor by technological operations, is suggested.

Key words: production line; subject of labor; production line; parameters of the state of the production line; technological position; transition period; production management systems; kinetic equation; mass production; work in progress; PDE production model.

Ref.: 27 titles.

Introduction. Variety and complexity of manufacturing technologies of a product creating premises to simulation of the technological process of production-technical system based on idea of it as a complex of the subjects of labor, which are in different stages of technological processing [1—4]. However, it is almost impossible to trace a behavior of the separately taken subject of the labor because of quality and the probable nature of influence on the subject of the labor of a technology equipment [4, 5]. Effective approach to simulation of big systems is the statistical approach [6—9], considering technological process at two levels of the description on the subject-technological level [10] and flow level [11]. At the subject-technological level patterns of behavior of separate elements of a system, on a flow level — their aggregated characteristics and communications between these characteristics are researched. Interconnection between levels is carried out through the kinetic equation [12—14]. This article is devoted to features of application of statistical approach to simulation of technological systems.

Main part. ***1. Subject-Technological Level of The Description of Technological Process.*** During execution of technological operation regarding work, the cost of technological resources by purposeful influence of a technology equipment is transferred [2; 5; 8; 15]. On each operation, inevitably there are oscillations of geometrical characteristics, physicomechanical properties of materials, which are caused by a complex of arbitrary and systematic production factors. Thus, technological process is accidental process of transition of subjects of the labor of one status in another because of influence of a technology equipment. Its status is defined as a status of number of subjects of the labor [5—9]. Coordinates in phase technological space to time point can provide the status of an subject of the labor (t, S, μ) [2; 6]: cost amount S_j ($) and intensity of transfer of expenses in unit of time μ_j ($ / hour) from a technology equipment on j subject of the labor, $0 < j < N$. Coordinates S_j and μ_j also define technological paths of subjects of the labor in phase technological space $S_j = S_j(t)$. Intensity of transmission of expenses $\Delta S = \Delta S(t)$ from means of labor on j subject of the labor for runtime of technological operation Δt is accidental process [4; 7; 8; 15], which value in the fixed time point is defined by a random variable [6]

$$\mu = \frac{\Delta S}{\Delta t}. \quad (1)$$

The system status in some time point will be defined if micro parameters are determined S_j and μ_j, and in any other time point, it is found from equations of state of parameters of an subject of the labor [16]:

$$\frac{dS_j}{dt} = \mu_j, \frac{d\mu_j}{dt} = f_j(t, S), \quad (2)$$

$f_j(t, S)$ — production function for a technology equipment [17]. If the quantity of subjects of the labor are much more than unit, then to solve system (2) from $2N$ — the equations are almost impossible. Instead of reviewing of a status of subject of the labor of technological process with parameters S_j and μ_j, we will enter the normalized function of distribution of subject of the labor on statuses. Each point in this space will set a status of an subject of the labor [6; 14; 18]. It is reasonable to expect that in case of big N this function will be well approximated by the continuous function of distribution of subject of the labor on statuses $\chi(t, S, \mu)$.

2. Kinetic Description of Technological Process. We will break phase space into such number of cells that the cell sizes $\Delta\Omega = \Delta S \cdot \Delta\mu$ there were much less values of characteristic (1) parameters of technological system and at the same time contained in themselves a large number of subjects of the labor. Instead of fixing exact parameter values of subjects of the labor, we will characterize approximately a status of technological system by number of subjects of the labor in each cell $\Delta\Omega$ [6; 8; 13]. If the sizes of a cell are rather small, then the approximate description will bear in itself almost so detailed information, as exact. That value $\chi(t, S, \mu) \cdot d\Omega$ represents number of subjects of the labor in an infinitesimal cell $\Delta\Omega$ phase technological space; we can on change of phase coordinate S and phase velocity μ, to judge also change of the function $\chi(t,S,\mu)$:

$$\frac{\partial\chi(t,S,\mu)}{\partial t}+\frac{\partial\chi(t,S,\mu)}{\partial S}\cdot\mu+\frac{\partial\chi(t,S,\mu)}{\partial\mu}\cdot f(S)=J(t,S,\mu),\ \frac{dS}{dt}=\mu;\frac{d\mu}{dt}=f(S). \quad (3)$$

The equation (3) describes change averaged on an infinitesimal cell of phase technological space $\Delta\Omega$ characteristics of subjects of the labor S_j, μ_j. We will read function $\chi(t,S,\mu)$ the normalized

$$\int_0^\infty dS\cdot\int_0^\infty d\mu\cdot\chi(t,S,\mu)=N. \quad (4)$$

Production function $f(t,S)$ is defined from the given method of production. In case of relocation along technological regarding work, impact from the equipment of the route located with a density is made $\lambda(S)$. We can speak only about probability that after such influence the subject of the labor will be in this or that status. This probable nature of influence can be considered; having set function $\psi(t,S,\mu)$, defining probability that after influence the subject of the labor will consume technological resources with intensity μ. Function $\psi(t,S,\mu)$ it is possible to set, analyzing passport data of a technology equipment:

$$\int_0^\infty\psi(t,S,\mu)\cdot d\mu=1,\ \int_0^\infty\mu^k\cdot\psi(t,S,\mu)\cdot d\mu=[\psi]_k,\ k=1,2\,\ldots. \quad (5)$$

Quantity of the subjects of the labor, which were affected in unit of time from a technology equipment in a cell $dS\cdot d\mu$ with coordinates (S,μ) and moved as a result of influence to a cell $dS\cdot d\tilde{\mu}$ with coordinates $(S,\tilde{\mu})$, in proportion to the work of a flow of subjects of the labor $\chi(t,S,\mu)\cdot\mu$ on transition probability $\psi(t,S,\tilde{\mu})\cdot d\tilde{\mu}$. Number of the subjects of the labor, which were affected in unit of time from a technology equipment and the accepted values in limits $(\tilde{\mu};\tilde{\mu}+d\tilde{\mu})$ there is a value $\psi(\tilde{\mu})\cdot\lambda(S)\cdot\mu\cdot\chi(t,S,\mu)\cdot d\tilde{\mu}\cdot dS\cdot d\mu$. Along with it in a volume, element $dS\cdot d\mu$ subjects of the labor from volume arrive $dS\cdot d\tilde{\mu}$ by the reverse transition in quantity $\psi(\mu)\cdot\lambda(S)\cdot\tilde{\mu}\cdot\chi(t,S,\tilde{\mu})\cdot d\tilde{\mu}\cdot dS\cdot d\mu$, and the total number of subjects of the labor in a volume element changes in unit of time on value $dS\cdot d\mu\cdot J$

$$J=\lambda(S)\cdot\int_0^\infty\left\{\psi(\mu)\cdot\tilde{\mu}\cdot\chi(t,S,\tilde{\mu})-\psi(\tilde{\mu})\cdot\mu\cdot\chi(t,S,\mu)\right\}d\tilde{\mu}. \quad (6)$$

From where the kinetic equation (3)—(6) can be presented in the form [12]

$$\frac{\partial\chi(t,S,\mu)}{\partial t}+\frac{\partial\chi(t,S,\mu)}{\partial S}\cdot\mu+\frac{\partial\chi(t,S,\mu)}{\partial\mu}\cdot f=\lambda\cdot\left\{\int_0^\infty\psi(\mu)\cdot\tilde{\mu}\cdot\chi(t,S,\tilde{\mu})d\tilde{\mu}-\mu\cdot\chi(t,S,\mu)\right\}. \quad (7)$$

In the majority practical cases equation (7) does not depend from a state of subjects of the labor before test of influence from the equipment from where

$$\frac{\partial\chi(t,S,\mu)}{\partial t}+\frac{\partial\chi(t,S,\mu)}{\partial S}\cdot\mu+\frac{\partial\chi(t,S,\mu)}{\partial\mu}\cdot f=\lambda(S)\cdot\left\{\psi(\mu)\cdot[\chi]_1-\mu\cdot\chi\right\}. \quad (8)$$

The solution of the equation (8) gives an opportunity to calculate values of flow parameters of technological process, is connected to great difficulties [6; 13; 19].

3. Flow Description of Technological Process. We will describe a status of technological process on a flow level the moments of a distribution function of subject of the labor on statuses $\chi(t,S,\mu)$

$$\int_0^\infty \mu^k \cdot \chi(t,S,\mu) d\mu = [\chi]_k, k = 0,1,2\dots. \quad (9)$$

It is known [1, 2, 7] that for the description of a status of production systems on a flow level use the two first the moment (9). The zero and first moments of a distribution function of subjects of the labor on statuses μ (9) have production interpretation: these are backlogs of subjects of the labor and their rate of movement along a technological chain [1; 2; 4]. Having increased the equation (8) on μ^k, $k = 0, 1, 2\dots$ and having integrated on all range μ, we will receive not closed equations of balances of a status of flow parameters of technological system [6; 20]:

$$\frac{\partial[\chi]_0}{\partial t} + \frac{\partial[\chi]_1}{\partial S} = \int_0^\infty d\mu \cdot J, \quad \frac{\partial[\chi]_k}{\partial t} + \frac{\partial[\chi]_{k+1}}{\partial S} = k \cdot f(t,S) \cdot [\chi]_{k-1} + \int_0^\infty d\mu \cdot \mu^k \cdot J, \quad k = 1, 2, 3\dots. \quad (10)$$

If average cost of resources $\langle \Delta S \rangle$, postponed during execution of technological operation regarding work the prime cost of the final product is much less S_d, what is characteristic of the technological process consisting of a large number of technological operations, the balance equations (10) in zero approximation in small parameter $\langle \Delta S \rangle / S_d << 1$ will take a form:

$$\frac{\partial[\chi]_0}{\partial t} + \frac{\partial[\chi]_1}{\partial S} = 0, \quad \frac{[\chi]_k}{[\chi]_1} = [\psi]_{k-1}, \quad \frac{\partial[\chi]_k}{\partial t} + \frac{\partial[\chi]_{k+1}}{\partial S} = k \cdot f(t,S) \cdot [\chi]_{k-1}, \quad k = 1, 2, 3\dots. \quad (11)$$

The system of the balance equations (11) is closed. For technological system, which macrostate is described by two parameters — a productions reserve of subject of the labor on technological and operation by their rate of movement, the system of the balance equations (12) can be written as:

$$\frac{\partial[\chi]_0}{\partial t} + \frac{\partial[\chi]_1}{\partial S} = 0, \quad \frac{[\chi]_2}{[\chi]_1} = [\psi]_1, \quad \frac{\partial[\chi]_1}{\partial t} + \frac{\partial[\chi]_2}{\partial S} = f(t,S) \cdot [\chi]_1. \quad (12)$$

The equations (12) describe a status of technological process through parameters — backlogs of subjects of the labor and their rate of movement on a process flow.

Conclusion. It is at the first sight would be possible to conclude that with increasing of a number of elements unimaginably increase complexity of technological system and not to find in her behavior also traces of any regularity [21; 22]. A research of the production- technical systems that consist of a very large number of the subjects of the labor, which are in technological process allowed to reveal important feature of such systems [4; 22]. It is the behavior of similar technological systems is defined by the regularities of special type which received names of statistical regularities. The importance of application of statistical approach essence in giving “the simplified mechanism” for the description of macroscopic characteristics of production-technical systems [23—27]. In many ways, which shows practical interest, such description is quite enough.

References

1. *Форрестер Дж.* Основы кибернетики предприятия / Дж. Форрестер. — М. : Прогресс, 1961. — 341 с.
2. Планирование дискретного производства в условиях АСУ / В. В. Шкурба [и др.]. — Киев : Техника, 1975. — 296 с.
3. *Вильсон, А. Дж.* Энтропийные методы моделирования сложных систем : пер. с англ. / А. Дж. Вильсон. — М. : Наука, 1978. — 248 с.
4. Теория моделей в процессах управления (Информационный и термодинамический аспекты) / Б. Н. Петров [и др.]. — М. : Наука, 1978. — 224 с.
5. *Летенко, В. А.* Организация, планирование и управление машиностроительным предприятием : в 2 ч. / В. А. Летенко, Б. Н. Родионов. — М. : Высш. шк., 1979. — Ч. 2 : Внутризаводское планирование. — 232 с.
6. *Пигнастый, О. М.* Статистическая теория производственных систем / О. М. Пигнастый. — Харків : ХНУ, 2007. — 388 с.
7. *Первозванский, А. А.* Математические методы в управлении производством / А. А. Первозванский. — М. : Наука, 1975. — 616 с.
8. *Власов, В. А.* Моделирование технологических процессов изготовления промышленной продукции / В. А. Власов, И. А. Тихомиров, И.И. Локтев. — Томск : Изд. ГТПУ, 2006. — 300 с.
9. *Bartholdi, J. J.* Deterministic chaos in a model of discrete manufacturing / J. J. Bartholdi, D. D. Eisenstein, Y. F. Lim // Naval Research Logistics. — 2009. — 56(4). — P. 293—299.
10. *Азаренков, Н. А.* Методы и модели предметно-технологического описания производственных систем / Н. А. Азаренков, О. М. Пигнастый, В. Д. Ходусов // Информационные технологии: наука, техника, технология, образование, здоровье : XXV Междунар. науч.-практ. конф. MicroCAD-2017 : Вестник НТУ «ХПИ». — 2017. — С. 344.
11. *Пигнастый, О. М.* Об основных направления исследования в статистической теории производственных систем / О. М. Пигнастый, В. Д. Ходусов // Комп`ютерне моделювання в наукоємних технологіях (КМНТ-2016) : зб. праць Міжнар. наук.-техн. конф. — Харків : ХНУ, 2016. — С. 325—328.
12. *Азаренков, Н. А.* Кинетическая теория колебаний параметров поточной линии / Н. А. Азаренков, О. М. Пигнастый, В. Д. Ходусов // Доповіді Нац. акад. наук України. — 2014. — № 12. — С. 36—43.
13. *Armbruster, D.* Thermalized kinetic and fluid models for reentrant supply chains. / D. Armbruster, C. Ringhofer // SIAM. Journal on Multiscale Modeling and Simulation. — 2005. — 3(4). — P. 782—800.
14. *Локтев, И. И.* Вопросы моделирования технологического процесса / И. И. Локтев // Изв. ГТПУ, 2005. — Т. 308, № 6. — С. 90—94.
15. *Пигнастый, О. М.* Стохастическая модель переноса технологических ресурсов на предмет труда в результате воздействия технологического оборудования. / О. М. Пигнастый // Научные ведомости Белгородского государственного университета. — 2016. — № 38/9. — С. 146—155.
16. *Пигнастый, О. М.* О построении целевой функции производственной системы / О. М. Пигнастый // Доповіді Нац. акад. наук України. — Київ : Академперіодика. — 2007. — № 5. — С. 50—55.
17. *Пигнастый, О. М.* Инженерно-производственная функция предприятия с серийным или массовым выпуском продукции / О. М. Пигнастый // Вопросы проектирования и производства конструкций летательных аппаратов. — Харьков : НАКУ. — 2005. — № 42(3). — С. 111—117.
18. *Якимович, С. Б.* Формирование комплектов машин лесозаготовок на основе распределения состояний предметов труда по маршруту технологических процессов. / С. Б. Якимович, И. Ю. Володина. — М. : МГУЛ, 2001. — № 2. — С. 128—136.
19. *Либов, Р.* Введение в теорию кинетических уравнений. / Р. Либов. — М. : Мир, 1974. — С. 372.
20. *Пигнастый, О. М.* Стохастическое описание экономико-производственных систем с массовым выпуском продукции / О. М. Пигнастый, В. П. Демуцкий, В. С. Пигнастая // Доповіді Нац. акад. наук України. — Київ : Академперіодика. — 2005. — № 7. — С. 66—71.
21. *Прыткин, Б. В.* Технико-экономический анализ производства. / Б. В. Прыткин. — М. : ЮНИТИ-ДАНА, 2000. — С. 399.
22. *Прангишвили, И. В.* Энтропийные и другие системные закономерности : Вопросы управления сложными системами / И. В. Прангишвили. — Ин-т проблем управления им. В. А. Трапезникова. — М. : Наука, 2003. — С. 428.
23. *Пигнастый, О. М.* Статистическое обоснование энтропийных закономерностей в моделях управления технологическими процессами. / О. М. Пигнастый, В. Д. Ходусов // Вісн. ХНУ. Сер. «Математическое моделирование. Информационные технологии. Автоматизированные системы управления». — 2012. — № 1037. — С. 168—174.
24. *Лысенко, Ю. Г.* Моделирование технологической гибкости производственно-экономических систем / Ю. Г. Лысенко, Н. В. Руменцев. — Донецк : ДонДУ, 2007. — С. 238.
25. *Бессонов, В. А.* Темповые производственные зависимости с ограниченным эффективным множеством / В. А. Бессонов, И. П Иванилов // Докл. Акад. наук СССР, 1989. — Т. 309, № 5. — С. 1033—1036.
26. *Lefeber, E.* Modeling, Validation and Control of Manufacturing Systems / E. Lefeber, R. A. Berg, J. E. Rooda // Proceeding of the 2004 American Control Conference. — Massachusetts, 2004. — P. 4583—4588.
27. *Berg, R.* Modelling and Control of a Manufacturing Flow Line using Partial Differential Equations /R. Berg, E. Lefeber, J. Rooda // IEEE. Transactions on Control Systems Technology. — 2008. — Vol. 16, № 1. — P. 130—136.

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